Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV (Old) EXAMINATION – WINTER 2019

Subject Code: 140001 **Date: 07/12/2019**

Subject Name: Mathematics-IV

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find the fifth root of unity. **07**

(b) Define interpolation. Using Lagrange's interpolation, find $y(2)$. **07**

x	-1	0	1	3
y	2	1	0	-1

Q.2 **(a)** Define bilinear transformation. Find bilinear transformation which maps points $0, 1, \infty$ into points $-1, -i, 1$ respectively. **07**

(b) Expand $\frac{1}{z^2-3z+2}$ in Laurent series in region (i) $1 < |z| < 2$ (ii) $|z| > 2$. **07**

OR

(b) Prove that $\tan h^{-1} z = \frac{1}{2} log\left(\frac{1+z}{1-z}\right)$ **07**

Q.3 **(a)** State Cauchy's integral formula. Evaluate $\oint_C \frac{3z^2+z}{z^2-1} dz$; where $|z-1| = 1$. **07**

(b) Define (i) Analytic function (ii) Harmonic function. **07**
If $f(z) = u(x,y) + iv(x,y)$ is analytic and $u(x,y) = y^3 - 3x^2y$, then find $v(x,y)$.

OR

Q.3 **(a)** Derive Cauchy – Riemann equations in polar form. **07**

(b) Evaluate $\oint_C \pi \exp(\pi z^2) dz$; where C is the boundary of square with vertices $0, 1, 1+i, i$ with counterclockwise direction. **07**

Q.4 **(a)** Prove that (i) $(1+\Delta)(1-\nabla) = 1$ (ii) $E = e^{hD}$ **07**

(b) Using Runge-Kutta 4th order method find $y(0.1)$. **07**

$$\frac{dy}{dx} = x^2 + y^2 \quad ; \quad y(0) = 1$$

OR

Q.4 **(a)** Find Newton's forward interpolating polynomial and hence find $y(5)$. **07**

x	4	6	8	10
y	1	3	8	16

(b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ (take h =1) using **07**

(i) Trapezoidal rule (ii) Simpson's 1/3 rule and (iii) Simpson's 3/8 rule.

Q.5 **(a)** Set up Newton iteration for computing square root of given positive number N and find $\sqrt{2}$. **07**

(b) Evaluate using 2 point and 3 point Gaussian integration. **07**

$$\int_0^1 \frac{dx}{1+x}$$

OR

Q.5 **(a)** Find a root of $x^3 - 4x - 9 = 0$ correct upto three decimal places using Bisection method. **07**

(b) Solve by Gauss-Siedel method correct upto 3 decimal places. **07**

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

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